From the parametric study for sensitivity considerations, the following observations emerge: a) If M is increased, the peak acceleration drops and if M is decreased, the peak acceleration goes up. b) If K is increased, the peak acceleration of M increases and if K is decreased, it drops down. c) If preload is decreased by 10%, there are variations in the peak acceleration. But if it is increased by 10%, the variations are not considerable. d) If the tire stiffness is increased, the peak acceleration goes up and if it is reduced, it comes down. e) When the velocity of descent is reduced, the peak acceleration drops down. f) In the case of m, if m is increased to 200% or decreased by 50% there is a variation in the peak acceleration; it drops down in the first case and increases in the second case.

V. Comparison

The dual-phase damping is better than constant or nonlinear orifice damping. The least value of the peak acceleration obtained is 30 ft/sec^2 for dual phase damping with the values of the parameters as $A=0.1$, $B=0.2$, $\zeta_1=0.9$ and $\zeta_2=0.1$. For constant damping, the least value is 36.9 ft/sec^2 where $\zeta=0.2$. With nonlinear orifice damping (where $F=C|\dot{x}|\dot{x}$) having C as 140 lbf/(ft/sec)^2, the peak acceleration is 114 ft/sec^2.

VI. Conclusion

From the previous results, it is observed that dual-phase damping serves better than constant or nonlinear orifice damping.

References

[1] J. C. Snowdon, "Isolation from Mechanical Shock with a Mounting System Having Nonlinear Dual-phase Damping," *The Shock and Vibration Bulletin*, Bulletin 41, Pt. 2, Dec. 1970.

[2] B. Milwitzky and F. E. Cook, "Analysis of Landing Gear Behavior," NACA Rept. 1154, 1953.

Note on Stability in Decelerating Flight

Peter R. Payne*
Payne, Inc., Annapolis, Md.

It is well known that the angle-of-attack oscillation of a re-entering space vehicle is very dependent upon whether the dynamic pressure q is increasing or decreasing. This effect is discussed, for example, in Refs. 1-7. The same type of phenomenon is observed when a bomb is dropped, or an escape system is ejected from an aircraft. In the latter case, a body which is found to be statically stable in the wind tunnel, and to have a positive aerodynamic damping, may appear to be unstable when launched from a rocket sled or an aircraft. That is, it experiences an angle-of-attack oscillation, the amplitude of which increases with time.

This particular problem can be explained by what is essentially a single-degree-of-freedom analysis which yields a closed-form solution. Consider the horizontal motion of a bluff body which has: a) no coupling out of the plane of oscillation, b) negligible variation of drag with angle-of-attack α, and c) negligible forces developed normal to its flight path. Assumption b) uncouples the velocity equation, which becomes

$$m\dot{u}+(C_D S)\ \tfrac{1}{2}\rho u^2=0$$

or

$$\dot{u}+\gamma u^2=0 \tag{1}$$

where $C_D S$ is the drag area, assumed constant; ρ is the mass density of the air and is assumed constant; u is the body's velocity; m is the body's mass; and $\gamma=(\rho C_D S/2m)$, a ballistic coefficient having units of length^{-1}. Integration of Eq. (1) gives

$$\frac{u}{u_o}=\frac{1}{1+\tau} \tag{2}$$

where the nondimensional time parameter $\tau=u_o\gamma t$ and u_o is the initial velocity.

The equation for pitching motion for the angle of attack is

$$I\ddot{\alpha}-C_{m_{\dot{\alpha}}}(\dot{\alpha}\ell/u)S\ell\tfrac{1}{2}\rho u^2-C_{m_\alpha}S\ell\tfrac{1}{2}\rho u^2\alpha=0 \tag{3}$$

where the symbols all have their usual meaning. This simplifies to

$$\ddot{\alpha}-\psi_1 u\dot{\alpha}-\psi_2 u^2\alpha=0 \tag{4}$$

where

$$\psi_1=(C_{m_{\dot{\alpha}}}S\ell^2\rho/2I)\quad(\text{length}^{-1})$$

$$\psi_2=(C_{m_\alpha}S\ell\rho/2I)\quad(\text{length}^{-2})$$

Substituting Eq. (2) for u, and expressing $\dot{\alpha}$ and $\ddot{\alpha}$ in terms of the nondimensional time parameter τ

$$\frac{d^2\alpha}{d\tau^2}-\frac{\psi_1}{\gamma}\frac{1}{(1+\tau)}\frac{d\alpha}{d\tau}-\frac{\psi_2}{\gamma^2}\frac{1}{(1+\tau)^2}\alpha=0 \tag{5}$$

The transformation $\eta=\log(1+\tau)$ leads to

$$\frac{d^2\alpha}{d\eta^2}-(1+\psi_1/\gamma)\frac{d\alpha}{d\eta}-\frac{\psi_2}{\gamma^2}\alpha=0 \tag{6}$$

This is a linear second-order differential in η, for which all the solutions are known. The lightly damped oscillatory solution will generally occur in practice, and for this case, the roots of the characteristic equation associated with Eq. (6) are

$$\lambda=m\pm in$$

where

$$m=\frac{1}{2\gamma}(\gamma+\psi_1)$$

$$n=\frac{1}{2\gamma}\sqrt{-[(\gamma+\psi_1)^2+\psi_2]} \tag{7}$$

$$\therefore\ \alpha=e^{m\eta}[A\cos n\eta+B\sin n\eta] \tag{8}$$

The arbitrary constants, A and B, are determined by the initial conditions at $\tau=0=\eta$; namely

$$\alpha=\alpha_0$$

$$\left[\frac{d\alpha}{d\eta}\right]_0=\left[\frac{d\alpha}{d\tau}\right]_0$$

which leads to

$$\frac{\alpha}{\alpha_0}=e^{m\eta}\left\{\cos n\eta+\frac{1}{n}\left[\frac{1}{\alpha_0}\left[\frac{d\alpha}{d\eta}\right]_0-m\right]\sin n\eta\right\} \tag{9}$$

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*President. Member AIAA.

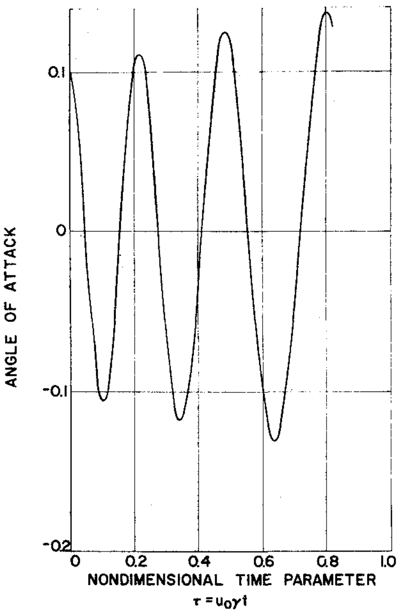

Fig. 1 Motion for P = 1000, and zero aerodynamic damping. $a_0 = 0.1$ **rads.**

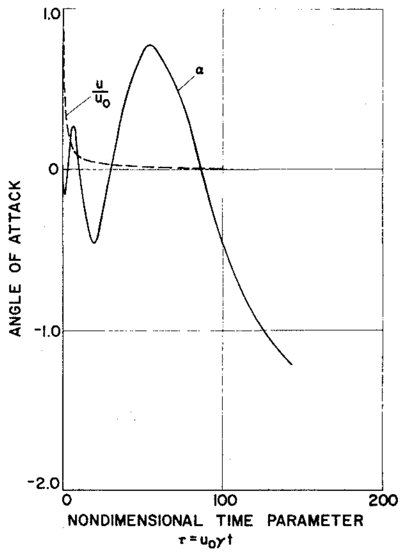

Fig. 2 Motion for P = 10 and zero aerodynamic damping. $a_0 = 0.1$ **rads.**

The motion will diverge or converge, depending on whether m is positive or negative. Note that

$$e^{m\eta} = e^{m\log(1+\tau)} = (1+\tau)^m = (1+\tau)^{(\frac{1}{2}+\psi_1/2\gamma)} \tag{10}$$

Thus the criteria for reducing amplitude is

$$1+(\psi_1/\gamma) < 0$$

or

$$(C_{m_{\dot\alpha}}/C_D) < -(r_G/\ell)^2 \tag{11}$$

where r_G is the radius of gyration ($I = mr_G^2$), and ℓ is the characteristic length on which the moment coefficients are based.

Suppose the speed drops to $u^* = zu_0\,(z<1)$. From Eq. (2), the associated time parameter is given by

$$z = 1/(1+\tau^*)$$

Thus from Eq. (1), the ratio

$$\frac{\alpha^*_{\max}}{\alpha_0} = z^{-\frac{1}{2}\,[1+(C_{m_{\dot\alpha}}/C_D)(\ell/r_G)^2]} \tag{12}$$

For example, if $C_{m_{\dot\alpha}} = 0$, then at half the initial speed ($z = 0.5$),

$$\alpha^*_{\max} = \sqrt{2}\,\alpha_0$$

This result is independent of the number of oscillatory cycles involved. That is, it is independent of the "aerodynamic stiffness."

The oscillatory frequency is

$$\Omega = \frac{d\theta}{dt} = \frac{d}{d\eta}(n\eta)\,\frac{d\eta}{d\tau}\,\frac{d\tau}{dt} = \frac{nu_0\gamma}{1+\tau}$$

$$= (\tfrac{1}{2}u_0\sqrt{-[(\gamma+\psi_1)^2+\psi_2]}\,/(1+u_0\gamma t) \tag{13}$$

so that instantaneous frequency is diminishing with time. If $C_{m_{\dot\alpha}} = 0$, the motion depends uniquely on τ and the dimensionless parameter

$$P = \frac{\psi_2}{\gamma^2} = \frac{C_{m_\alpha}}{C_D^2}\cdot\frac{2\ell m}{\rho S r_G^2} \tag{14}$$

Solutions for $P = 1000$ and 10 are plotted in Figs. 1 and 2.

References

[1] Friedrich, H.R. and Dore, F.J., "The Dynamic Motion of a Missile Descending Through the Atmosphere," *Journal of Aeronautical Sciences,* Sept. 1955, pp. 628-638.

[2] Allen, H.J., "Motion of a Ballistic Missile Angularly Misaligned with the Flight Path Upon Entering the Atmosphere and its Effect upon Aerodynamic Heating, Aerodynamic Loads, and Miss Distance," NACA TN 4048, Oct. 1957.

[3] Allen, H.J. and Eggers, A.J., Jr., "A Study of the Motion and Aerodynamic Heating of Ballistic Missiles Entering the Earth's Atmosphere at High Supersonic Speeds," NACA Rept. 1381, 1958.

[4] Tobak, J. and Allen, H.J., "Dynamic Stability of Vehicles Traversing Ascending or Descending Paths Through the Atmosphere," NACA TN 4275, July 1958.

[5] Laitone, E.V., "Dynamic Longitudinal Stability Equations for Re-Entry Ballistic Missile," *Journal of the Aero/Space Sciences,* Feb. 1958, pp. 94-98.

[6] Lichtenstein, J.H., "Analytical Investigation of the Dynamic Behavior of a Nonlifting Manned Re-Entry Vehicle," NASA TN D-416, Sept. 1960.

[7] Tobak, M., "Analytical Study of the Tumbling Motions of Vehicles Entering Planetary Atomspheres," NASA TN D-1549, Oct. 1962.

Large Fracture Toughness Boron-Epoxy Composites

A.G. Atkins*

University of Michigan, Ann Arbor, Mich.

IN conventional brittle fiber/brittle filament composites, when the interfacial bond between filament and matrix is strong, fracture is often caused by rapid matrix cracks which break through all filaments in their paths. The toughness of such composites is low because, in general, the critical transfer length associated with strong interfacial bonding is small, which limits various components of total toughness—the 'surfaces' component, Piggott-Fitz-Randolph stress redistribution and Cottrell-Kelly pull-out (see, for

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*Associate Professor of Mechanical Engineering.